Constructing Schedules for Sports Leagues with Divisional and Round-robin Tournament Play

Jeffrey Larson Mikael Johansson
KTH Automatic Control Group
OCTOBER 29, 2013

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Thanks to Lars Westman with the Swedish Handball Federation

Background

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  M.S., Ph.D. in Applied Mathematics - University of Colorado Denver

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  - Derivative-free Optimization
  - Truck Platooning

Contents

Elitserien – Top Level of Swedish Handball

- 14-team league; owners want more than 26 games, but not 39 games
- Form 2 divisions which hold a single round-robin tournament
- Has standard requirements, so hopefully the results are useful
- Want a very fair home-away patterns in their schedule
- Desire a template which they can use on their own

Template

0	-2	3	-4	5	-6	7	-8	9	-5	6	-7	10	-11	12	-13	4	-14	2	-3	...
-7	1	0	-3	4	-5	6	-9	7	-12	10	-6	5	-4	11	-14	13	3	-1	8	...
-6	7	-1	2	0	-4	5	-10	8	-7	9	-5	11	-13	6	-12	14	-2	4	1	...
-5	6	-7	1	-2	3	0	-11	10	-6	5	-13	12	2	-14	8	-1	9	-3	7	...
4	0	-6	7	-1	2	-3	13	-14	1	-4	3	-2	8	-7	9	-10	-11	6	-12	...
3	-4	5	0	-7	1	-2	12	-13	4	-1	2	-8	14	-3	7	-9	10	-5	-11	...
2	-3	4	-5	6	0	-1	14	-2	3	-8	1	-9	-10	5	-6	11	-12	13	-4	...

0	9	-10	11	-12	13	-14	1	-3	-9	7	-11	6	-5	10	-4	12	-13	14	-2	...
14	-8	0	10	-11	12	-13	2	-1	8	-3	-10	7	-12	13	-5	6	-4	11	-14	...
13	-14	8	-9	0	11	-12	3	-4	14	-2	9	-1	7	-8	-11	5	-6	12	-13	...
12	-13	14	-8	9	-10	0	4	-12	13	-14	8	-3	1	-2	10	-7	5	-9	6	...
-11	0	13	-14	8	-9	10	-6	11	2	-13	14	-4	9	-1	3	-8	7	-10	5	...
-10	11	-12	0	14	-8	9	-5	6	-11	12	4	-14	3	-9	1	-2	8	-7	10	...
-9	10	-11	12	-13	0	8	-7	5	-10	11	-12	13	-6	4	2	-3	1	-8	9	...

Background

There has been some work with strength groups (teams of similar ability) by Briskorn

- Group-balanced schedules: No team plays a team from the same group in p periods
- Group-changing schedules: No team plays a team from the same group in consecutive periods

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- Group-balanced schedules: No team plays a team from the same group in p periods
- Group-changing schedules: No team plays a team from the same group in consecutive periods

No work in the literature addresses desirable home-away patterns for teams in different groups

Home/Away Pattern Sets

- General scheduling is very hard
- A common simplifying method involves constructing home/away pattern (HAP) sets
- Desirable home-away patterns for each team

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Team 1	AHAHA
Team 2	AAHAH
Team 3	AHHAH
Team 4	HAHAH
Team 5	HHAHA
Team 6	HAAHA

Home/Away Pattern Sets

- General scheduling is very hard
- A common simplifying method involves constructing home/away pattern (HAP) sets
- Desirable home-away patterns for each team

Team 1	AHAHA
Team 2	AAHAH
Team 3	AHHAH
Team 4	HAHAH
Team 5	HHAHA
Team 6	HAAHA

But not every HAP set is schedulable

List of Requirements

1. Each 7-team division must hold a SRRT to start the season.
2. This must be followed by two SRRTs between the entire league, the second SRRT being a mirror of the first.
3. There must be a minimum number of breaks in the schedule.
4. Each team has one bye during the season (to occur during the divisional RRT).
5. At no point during the season can the number of home and away games played by a team differ by more than 1.
6. Any pair of teams must have consecutive meetings occur at different venues. (AVR)
7. Each division must have 3 pairs of complementary schedules.

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1. Each 7-team division must hold a SRRT to start the season.
2. If you end the first SRRT: away at team 1 and home for team 2, you start the next SRRT: away at team 2 and home for team 1
3. There must be a minimum number of breaks in the schedule.
4. Each team has one bye during the season (to occur during the divisional RRT).
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2. This must be followed by two SRRTs between the entire league, the second SRRT being a mirror of the first.
3. We don't want "HH" or "AA"
4. Each team has one bye during the season (to occur during the divisional RRT).
5. At no point during the season can the number of home and away games played by a team differ by more than 1.
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2. This must be followed by two SRRTs between the entire league, the second SRRT being a mirror of the first.
3. There must be a minimum number of breaks in the schedule.
4. Each division has an odd number of teams
5. At no point during the season can the number of home and away games played by a team differ by more than 1.
6. Any pair of teams must have consecutive meetings occur at different venues. (AVR)
7. Each division must have 3 pairs of complementary schedules.

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4. Each team has one bye during the season (to occur during the divisional RRT).
5. Can't start "AHAAH"
6. Any pair of teams must have consecutive meetings occur at different venues. (AVR)
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4. Each team has one bye during the season (to occur during the divisional RRT).
5. At no point during the season can the number of home and away games played by a team differ by more than 1.
6. Teams meeting 3 times: “AHA” or “HAH” (not “AAH” or ”HHA”)
7. Each division must have 3 pairs of complementary schedules.

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6. Any pair of teams must have consecutive meetings occur at different venues. (AVR)
7. A H A H B A H A H A H A H H A H A H A H ...
   H A H B A H A H A H A H A A H A H A H A ...

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Previous Results

Two results from the literature apply

- Every n-team RRT, n even, must have at least $n-2$ breaks, DeWerra (1981)
- For an n-team RRT, n odd, there exists a unique no break tournament, Fronček (2005)

Previous Results

BAHAHAH
HBAHAHA
AHBAHAH
HAHBAHA
AHAHBAH
HAHAHBA
AHAHAHB

or

BHAHAHA
ABHAHAH
HABHAHA
AHABHAH
HAHABHA
AHAHABH
HAHAHAB

AHAHAHAHAHAHA
AHAHAHAHAHAHH
AHAHAHAHAHHAH
AHAHAHAHHAHAH
AHAHAHHAHAHAH
AHAHHAHAHAHAH
AHHAHAHAHAHAH
HAAHAHAHAHAHA
HAHAAHAHAHAHA
HAHAHAAHAHAHA
HAHAHAHAAHAHA
HAHAHAHAHAAHA
HAHAHAHAHAHAA
HAHAHAHAHAHAH

Constructing HAP Sets

B	H	A	H	A	H	A	H	A	H	A	A	H	A	H	A	H	A	H	A	H	A	...
H	A	B	H	A	H	A	H	A	H	A	H	A	H	A	A	H	A	H	A	H	A	...
H	A	H	A	B	H	A	H	A	A	H	A	H	A	H	A	H	A	H	A	H	A	...
H	A	H	A	H	A	B	H	A	H	A	H	A	H	A	H	A	A	H	A	H	A	...
A	B	H	A	H	A	H	A	H	A	H	A	H	A	H	H	A	H	A	H	A	H	...
A	H	A	B	H	A	H	A	H	H	A	H	A	H	A	H	A	H	A	H	A	H	...
A	H	A	H	A	B	H	A	H	A	H	A	H	A	H	A	H	H	A	H	A	H	...
B	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	H	H	A	H	...
A	H	B	A	H	A	H	A	H	A	H	H	A	H	A	H	A	H	A	H	A	H	...
A	H	A	H	B	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	...
A	H	A	H	A	H	B	A	H	A	H	A	H	H	A	H	A	H	A	H	A	H	...
H	B	A	H	A	H	A	H	A	H	A	H	A	A	H	A	H	A	H	A	H	A	...
H	A	H	B	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	A	H	A	...
H	A	H	A	H	B	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	H	...

Constraints

1. Each 7-team division must hold a SRRT to start the season.
2. This must be followed by two SRRTs between the entire league, the second SRRT being a mirror of the first.
3. There must be a minimum number of breaks in the schedule.
4. Each team has one bye during the season (to occur during the divisional RRT).
5. At no point during the season can the number of home and away games played by a team differ by more than 1.
6. Any pair of teams must have consecutive meetings occur at different venues. (AVR)
7. Each division must have 3 pairs of complementary schedules.

Counting HAP Sets

Proposition

For an n-team tournament, $\frac{n}{2}$ odd, with a divisional RRT before full-league DRRT, there are

$$ {}_{\frac{n}{2}}P_{\frac{n-2}{4}} \times \left(\frac{n+2}{4}\right)^3 \times \frac{n-2}{4}! $$

unique HAP sets satisfying the requirements, except possibly for the AVR, with $\frac{n-2}{4}$ pairs of complementary schedules within each division.

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unique HAP sets satisfying the requirements, except possibly for the AVR, with $\frac{n-2}{4}$ pairs of complementary schedules within each division.

For the 14-team Elitserien, this is 80640 HAP sets.

Example Violating AVR

By construction, every HAP Set can be scheduled in a manner satisfying the League Requirements, except possibly for the AVR.

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By construction, every HAP Set can be scheduled in a manner satisfying the League Requirements, except possibly for the AVR.

H	A	H	B	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	A
H	A	H	A	H	B	A	H	A	H	A	H	A	H	A	H	A	H	A	H

Simple Condition

For an arbitrary HAP set S, define

$$S(t,p) = \begin{cases} H: & \text{if team } t \text{ plays home in period } p, \\ A: & \text{if team } t \text{ plays away in period } p, \\ B: & \text{if team } t \text{ has a bye in period } p. \end{cases}$$

Simple Condition

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Proposition

For a HAP set S to be schedulable, for any two teams t_1 and t_2 in the same division, there must be two periods p_1 in Part I and p_2 in Part II such that

$$\begin{aligned} S(t_1,p_1) = H \quad &\textit{and} \quad S(t_2,p_1) = A, \\ S(t_1,p_2) = A \quad &\textit{and} \quad S(t_2,p_2) = H. \end{aligned}$$

Efficiency of Simple Test

n	HAP sets	HAP removed by simple condition	% removed
6	24	8 (of 20 unschedulable)	40%
10	1080	396 (of 998 unschedulable)	$\approx$ 40%
14	80640	30720 (of 79024 unschedulable)	$\approx$ 39%

Another Necessary Condition

B	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A	H	H
A	H	B	A	H	A	H	A	H	A	H	H	A	H	A	H	A	H	A	H
A	H	A	H	B	A	H	A	H	A	H	A	H	A	H	A	H	A	H	A

Another Necessary Condition

- Check if i or j is already "committed" to play another team in every period when they could possibly meet.
  - For example, if i can only play j in periods p_1 or p_2
  - i must play k_1 in p_1
  - j must play k_2 in p_2
- This is only slightly more expensive computationally to check than the simple condition, but it catches many "deeper" contradictions.
- This condition removes 46944 of the 80640 HAP sets (59%).

Latin-square Condition

Build an $n \times n$ array, where each entry (i, j) is a vector of periods when it is possible for teams i and j to meet.

- If (i, j) has only one entry, remove that value (if possible) from any vector (i, k), $k \neq j$ and any vector (k, j), $k \neq i$.
- If (i, j) has more than one entry, see if any value is unique in a row or column. Replace (i, j) by that value.
- Stop if any (i, j) is empty, or no change is observed after checking the above two conditions for all (i, j).

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- Stop if any (i, j) is empty, or no change is observed after checking the above two conditions for all (i, j).

Three things can happen

1. An entry (i, j) of the Latin square becomes an empty vector
2. Every entry of the Latin square becomes a 1-dimensional vector
3. The Latin square remains unchanged after checking every entry (i, j)

Latin Square Example

Team 1	AHAHA
Team 2	AAHAH
Team 3	AHHAH
Team 4	HAHAH
Team 5	HHAHA
Team 6	HAAHA

Latin Square Example

Team 1	AHAHA
Team 2	AAHAH
Team 3	AHHAH
Team 4	HAHAH
Team 5	HHAHA
Team 6	HAAHA

	1	2	3	4	5	6
1						
2	[2,3,4,5]					
3	[3,4,5]	[2]				
4	[1,2,3,4,5]	[1]	[1,2]			
5	[1]	[1,2,3,4,5]	[1,3,4,5]	[2,3,4,5]		
6	[1,2]	[1,3,4,5]	[1,2,3,4,5]	[3,4,5]	[2]	

Efficiency of the Latin Square Approach

n	HAP sets	HAP removed by L.S. condition	% removed
6	24	20 (of 20 unschedulable)	100%
10	1080	998 (of 998 unschedulable)	100%
14	80640	75995 (of 79024 unschedulable)	$\approx 96\%$

Efficiency of the Latin Square Approach

n	HAP sets	HAP removed by L.S. condition	% removed
6	24	20 (of 20 unschedulable)	100%
10	1080	998 (of 998 unschedulable)	100%
14	80640	75995 (of 79024 unschedulable)	$\approx$ 96%

n	HAP sets	HAP removed "one pass"	% removed
6	24	10 (of 20 unschedulable)	50%
10	1080	504 (of 998 unschedulable)	$\approx$ 51%
14	80640	51946 (of 79024 unschedulable)	$\approx$ 66%

Final Template

0	-2	3	-4	5	-6	7	-8	9	-5	6	-7	10	-11	12	-13	4	-14	2	-3	...
-7	1	0	-3	4	-5	6	-9	7	-12	10	-6	5	-4	11	-14	13	3	-1	8	...
-6	7	-1	2	0	-4	5	-10	8	-7	9	-5	11	-13	6	-12	14	-2	4	1	...
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2	-3	4	-5	6	0	-1	14	-2	3	-8	1	-9	-10	5	-6	11	-12	13	-4	...

0	9	-10	11	-12	13	-14	1	-3	-9	7	-11	6	-5	10	-4	12	-13	14	-2	...
14	-8	0	10	-11	12	-13	2	-1	8	-3	-10	7	-12	13	-5	6	-4	11	-14	...
13	-14	8	-9	0	11	-12	3	-4	14	-2	9	-1	7	-8	-11	5	-6	12	-13	...
12	-13	14	-8	9	-10	0	4	-12	13	-14	8	-3	1	-2	10	-7	5	-9	6	...
-11	0	13	-14	8	-9	10	-6	11	2	-13	14	-4	9	-1	3	-8	7	-10	5	...
-10	11	-12	0	14	-8	9	-5	6	-11	12	4	-14	3	-9	1	-2	8	-7	10	...
-9	10	-11	12	-13	0	8	-7	5	-10	11	-12	13	-6	4	2	-3	1	-8	9	...

Additional Desires

Additional requests and concerns can be addressed when assigning teams to numbers:

- Venue availabilities
- Desired derby games
- More meetings between the top teams and between the bottom teams in the last weeks.

Summary

- We constructed (and count) HAP sets with a minimum number of breaks
- We are able to remove many HAP sets as unschedulable with respect to the AVR
- We can then construct a template which can be agreed upon by the league owners
- We assign teams to numbers to construct a yearly schedule

Thanks

Questions?

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